Short-Term Reservoir Storage Frequency Relationships

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Abstract: The water rights analysis package (WRAP) is a generalized river/reservoir system simulation model that is routinely applied in Texas in regional and statewide planning studies and administration of the water right permit system. The WRAP modeling system was recently expanded by adding short-term storage frequency and supply reliability analysis capabilities. Individual reservoirs and multiple-reservoir systems can be analyzed considering numerous water users and complex water management practices. The new modeling features are based on dividing the hydrologic period-of-analysis into many short-simulation sequences with each starting with the same storage conditions. Two alternative frequency/reliability analysis methodologies, called the equal-weight and probability-array options, are compared in this paper with a case-study application. The probability array option is designed to improve the accuracy of storage frequency estimates by modeling hydrologic persistence as reflected in the preceding reservoir storage contents on the basis of a regression of natural streamflow versus preceding storage from a long-term simulation. **DOI:** 10.1061/(ASCE)WR.1943-5452.0000218. *© 2012 American Society of Civil Engineers.*

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Introduction

Short-term storage frequency and supply reliability analysis capabilities were recently implemented in the water rights analysis package (WRAP) modeling system to support drought management and operational planning activities. WRAP was originally developed and is routinely applied for long-term planning studies and evaluation of water right permit applications. The new short-term conditional reliability modeling (CRM) methodologies outlined in this paper use the same input data sets as conventional long-term WRAP applications. However, the hydrologic period-of-analysis is subdivided into many short simulation sequences. Simulations are repeated with each sequence of naturalized streamflows and net reservoir surface evaporation minus precipitation rates, with each simulation starting with the same initial reservoir storage conditions. Frequency and reliability analyses of the simulation results are performed. A variety of water supply and hydropower reliability metrics and streamflow and reservoir storage frequency relationships are generated with the WRAP modeling system with either the new short-term CRM or conventional long-term analyses. This paper focuses on developing storage frequency relationships conditioned on preceding reservoir storage contents.

Two alternative CRM strategies, called the equal-weight and probability-array options, are incorporated in WRAP. The difference between the two methods is the approach adopted for assigning probabilities to each hydrologic simulation sequence for use in frequency and reliability analyses. In the equal-weight method, each of the short-term simulations are weighted the same in the frequency analyses. The probability-array approach uses a set of computational techniques that assigns varying probabilities to the hydrologic sequences. The equal-weight approach provides a significant advantage in that it is simpler. The probability-array option adds complexity but may improve the accuracy of the analyses if a significant correlation between naturalized streamflow and preceding storage can be derived from a long-term simulation.

Water Rights Analysis Package Modeling System

The Texas Commission on Environmental Quality (TCEQ) maintains a water availability modeling (WAM) system used in administration of the water rights permit system, regional and statewide planning, and other activities (Wurbs 2005). The TCEQ WAM system consists of the generalized WRAP river/reservoir system simulation model and hydrology and water rights input files for the 23 river basins of Texas. These WRAP input data sets model approximately 3,450 reservoirs, 8,000 water right permits, five interstate compacts, and an international treaty.

WRAP simulates water resources development, management, and use in a river basin or multiple-basin region under priority-based water allocation systems. The generalized model facilitates assessments of water availability and reliability for municipal, industrial, and agricultural water supply, hydroelectric energy generation, environmental instream flows, and reservoir storage. Basin-wide effects of water development projects and management practices are modeled. The WRAP simulation model performs the river/reservoir system water allocation simulation for a hydrologic period-of-analysis for any number of years by using a monthly time step. A WRAP post-simulation program organizes simulation results and develops frequency relationships, reliability indices, and summary statistics.

WRAP is generalized for application to river/reservoir systems located anywhere in the world, with input data sets developed for particular river basins of concern. For studies in Texas, TCEQ

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WAM system input files are altered as appropriate to reflect proposed water management plans of interest. WRAP and its application in the TCEQ WAM system are described by Wurbs (2005). The public domain software and documentation (Wurbs 2011a, b; Wurbs and Hoffpauir 2011) are available at http://ceprofs.tamu.edu/rwurbs/wrap.htm, which connects with the TCEQ WAM website that provides data sets for Texas river basins and other information.

Alternative Modeling Approaches

Wurbs (1996) and Nagy et al. (2002) review models for developing the probability distribution of reservoir storage for specified initial storage conditions. These reviews date back to the Moran (1954) model that computes the probability distribution of storage at the end of consecutive years assuming independent annual inflows and constant reservoir outflows and losses. Gould (1961) addressed some of the limitations of the Moran model by deriving the transition matrix for a monthly time-step while considering variations in various factors affecting the storage budget. Klemes (1981) and McMahon and Mein (1986) outline computational methods, cite many references, and highlight shortcomings of the early stochastic storage probability models.

The Tennessee Valley Authority (Gilbert and Shane 1982) pioneered the use of models on the basis of computing frequency statistics from the results of many simulations of a complex reservoir system, with each simulation starting with the same initial storage contents. The R. J. Brandes Company (1998) performed CRM for the Amistad and Falcon Reservoirs on the Rio Grande River by adding repetitive multiple-simulation features to a long-term simulation model developed by the Texas Water Development Board. Vaugh and Maidment (1987) adopted the term transient analysis for this general approach and compared it to the Gould (1961) probability matrix model by applying both methods to reservoirs on the Colorado River in Texas. The WRAP equal-weight CRM option combines this general approach with the comprehensive generalized modeling capabilities provided in the public domain WRAP modeling system.

Another approach adopted by many investigators is on the basis of generating an ensemble of streamflow forecasts by multiple watershed precipitation-runoff model simulations with different sequences of precipitation, temperature, and other climatic inputs. Each simulation begins with the same specified soil moisture and other parameters characterizing initial watershed conditions. The National Weather Service has developed an ensemble streamflow prediction (ESP) system that uses a watershed model with historical precipitation and climatic input data to generate a set of possible streamflow scenarios conditioned on the initial state of a particular river basin at a particular time (Day 1985; Franz et al. 2003). Ensembles of streamflow scenarios also have been generated by using precipitation-runoff models in France (Thirel et al. 2010), Australia (Wang et al. 2011), and elsewhere. Alemu et al. (2011) illustrate the incorporation of ensemble streamflow forecasts in models for optimizing reservoir operations.

The TCEQ WAM system is on the basis of naturalized monthly streamflow data sets, representing natural historical river basin hydrology, generated by adjusting gauged flows to remove the effects of river basin development and water use. The WAM system includes naturalized monthly flows covering periods longer than 50 years for approximately 500 gauged sites. The TCEQ and its partner agencies and contractors considered the role of precipitation-runoff models in developing sequences of monthly naturalized flows during the development of the WAM system. However, the approach of adjusting gauged flows was adopted because it is generally more accurate and requires less effort than computing flows with a precipitation-runoff model for the statewide WAM system. The new CRM component of WRAP is designed to use the available statewide data set of naturalized flows and net reservoir surface evaporation minus precipitation rates.

Climate change is recognized as a concern in applying the WAM system. Wurbs et al. (2005) applied a watershed model with precipitation and temperature data representing alternative future climate scenarios, which were generated with a global circulation model that explored effects on the WAM naturalized flow data set for the Brazos River Basin. Modeling the effects of future climate change is highly uncertain. However, long-term future climate change is a greater concern for long-term planning applications of the WRAP/WAM system than for short-term CRM.

Analyses indicate no long-term trends in the 1900–2007 naturalized flows in the Brazos River Basin used in the case study presented in this paper. The effects of El Nino and La Nina cycles are inherent in the naturalized flows and thus are somewhat reflected in the simulated reservoir storage volumes used in the probability array CRM methodology.

The basic idea of the probability array option presented in this paper is that simulated reservoir storage for a specified scenario provides an index of preceding hydrologic conditions. Lower naturalized flows are more likely to follow low rather than high reservoir storage levels. Salazar (2002) and Salazar and Wurbs (2004) developed supply reliability indices on the basis of a conditional frequency duration curve for establishing probability distributions for naturalized flows conditioned on preceding storage conditions as reflected in discrete storage intervals. This approach was determined to be difficult to apply in practical applications and was never fully implemented in WRAP. Continuing research by Olmos (2004) and Schnier (2010) contributed to the probability-array methodology that was recently implemented in WRAP and described in this paper.

WRAP Conditional Reliability Modeling

Conditional reliability modeling consists of estimating supply reliability and storage frequency metrics conditioned on preceding storage. The terms CRM and short-term modeling are used interchangeably. The likelihood of meeting reservoir storage, water supply, instream flow, and hydroelectric energy targets during the next month to next year or perhaps longer is assessed as a function of the preceding reservoir storage contents along with all the data reflected in conventional long-term WRAP modeling. Applications of CRM include developing reservoir system operating rules and drought management plans, operational planning studies, administering water right permits and water supply contracts, and decision-support during drought.

A conventional simulation is performed for the entire hydrologic period-of-analysis with initial reservoir storage contents specified only for the beginning of the first month. In CRM, the hydrologic period-of-analysis is subdivided into many shorter sequences, and the same specified initial reservoir storage volumes are reset at the beginning of each simulation sequence.

CRM uses the same simulation input data sets as long-term applications. The naturalized flows and net reservoir surface evaporation minus precipitation rates are subdivided within the model into many short-simulation sequences. The modeling system provides various options for dividing the hydrologic period-of-analysis into multiple shorter sequences, assigning probabilities to the

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1. Equal-Weight Strategy
   choice of annual or monthly cycle options
2. Probability Array Strategy
   choice of annual or monthly cycle options
   selection of location and number of months for flows
   choice of regression equation
   log-normal distribution or Weibull relative frequency

Fig. 1. Outline of conditional reliability modeling computational options

sequences, and developing water supply reliability and reservoir storage frequency relationships from simulation results.

Choices in formulating a CRM application discussed in this paper are outlined in Fig. 1. The simulation is identical with either the equal-weight or probability-array options, but reliability and frequency analyses of simulation results differ significantly.

Two alternative approaches, called the annual and monthly cycle options, are provided for dividing the hydrologic period-of-analysis into simulation sequences. The hydrologic simulation sequence length may be any integer number of months with either option. The annual cycle option starts each simulation sequence in the same specified month of the year. With the monthly cycle option, each simulation begins in the next month after the starting month of the preceding simulation. The annual cycle captures seasonality, but the number of simulations is limited to the number of years in the total period of analysis or less. The monthly cycle allows up to 12 times more simulations than the annual cycle option, but at the loss of seasonality.

The accuracy of frequency and reliability estimates depends both on properly modeling seasonal characteristics of hydrology and maximizing the number of hydrologic sequences used in the analyses. The case study has a 108-year hydrologic period-of-analysis extending from January 1900 through December 2007. The annual cycle option with a 12-month simulation period starting in July is adopted, resulting in 107 simulation sequences. The monthly cycle option results in 1,285 twelve-month simulation sequences but does not capture seasonality. The annual cycle option was selected for the case study because streamflows vary greatly seasonally, and 107 simulations are considered to be a large number of hydrologic sequences.

Brazos River Basin Data Sets

The 118,000 km^2 Brazos River Basin extends from New Mexico southeasterly across Texas to the Gulf of Mexico. Climate, vegetation, topography, land use, and water use vary greatly across the basin. Mean annual precipitation varies from 48 cm in the upper basin that lies in the High Plains to 115 cm in the lower basin in the Gulf Coast Region.

The Brazos River Basin served as a case study for developing, testing, and applying CRM capabilities (Wurbs et al. 2011). A condensed version of the TCEQ WAM system data set (Wurbs and Kim 2008) designed for studies of the operation of the Brazos River Authority (BRA) reservoir system was adopted for CRM. The BRA is applying the model during the 2011 drought to assess the likelihood of reservoirs remaining at various levels during future months. Probability estimates are updated as storage levels change during the drought.

The TCEQ WAM system contains WRAP input data sets for each of the river basins of Texas for two alternative water use scenarios, authorized and current. The authorized use scenario is on the basis of the premise that all water right permit holders appropriate the full amount of water legally authorized by their permits. The current use scenario represents a best estimate of actual water use in recent years, which is significantly less than authorized use for the Brazos River Basin. Water demands are projected to increase in the future.

The BRA and more than 1,000 water districts, cities, companies, and individuals hold water right permits to use the waters of the Brazos River and its tributaries. The Brazos River Basin data set in the TCEQ WAM system contains 711 reservoirs, 77 primary control points in which naturalized flows are provided as an input file, and more than 3,000 secondary control points in which naturalized flows are synthesized as the simulation model is executed. Most of the 77 primary control points are gaging stations. The secondary control points are sites of dams, diversions, return flows, instream flow targets, and stream confluences.

The large and complex Brazos WAM data set is necessary for planning and permission of water right applications for which the WAM system was developed, and it can be used directly for CRM. However, a much simpler model focused on the BRA reservoir system facilities and BRA operational planning studies, which were the primary motivation for the Brazos CRM analyses. Wurbs and Kim (2008) condensed the WAM system data set to focus on management of the BRA system. Most of the water rights, control points, and reservoirs are removed with their effects retained in the adjusted available stream flow provided as input in the condensed data set.

The Brazos River Authority condensed (BRAC) data set contains 48 control points and 14 reservoirs as shown in Table 1 and Fig. 2. The available streamflows at the 48 control points that were provided as input to the simulation model are the flows that

Table 1. Reservoirs in the BRAC Model

Reservoir	Owner	Stream	Watershed area (km^2)	Conservation capacity (10^6 m^3)	Flood control (10^6 m^3)
Proctor	USACE/BRA	Leon River	3,270	67.5	383
Belton	USACE/BRA	Leon River	9,170	534	790
Stillhouse Hollow	USACE/BRA	Lampasas River	3,420	277	482
Georgetown	USACE/BRA	San Gabriel R.	642	45.6	108
Granger	USACE/BRA	San Gabriel R.	1,910	62.3	200
Possum Kingdom	BRA	Brazos River	61,700	681	—
Granbury	BRA	Brazos River	66,900	164	—
Whitney	USACE/BRA	Brazos River	70,600	692	1,693
Aquilla	USACE/BRA	Aquilla Creek	2,510	51.4	107
Waco	USACE/BRA	Bosque River	4,290	255	683
Limestone	BRA	Navasota River	1,750	257	—
Somerville	USACE/BRA	Yequa Creek	2,610	190	417
Hubbard Creek	WCTMWD	Hubbard Creek	2,820	392	—
Squaw Creek	CPPP	Squaw Creek	—	186	—

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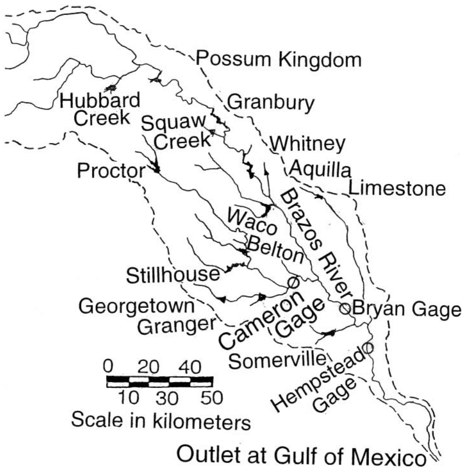

Fig. 2. Selected 14 reservoirs in the Brazos River Basin including five reservoirs above the Cameron gauge on the Little River

were available to the BRA, the West Central Texas Municipal Water District (WCTMWD), and the Comanche Peak Power Plant (CPPP), and considered the effects of all the other permitted water users in the basin including the 697 other smaller reservoirs. For CRM performed with a full WAM data set, hydrologic sequences are comprised of naturalized flows. However, in adopting the condensed BRAC data set, the CRM hydrologic simulation sequences are the adjusted streamflows legally available to the water management agencies that operate the 14 major reservoirs.

The CRM simulations discussed in this paper were performed with a version of the BRAC data set that reflects actual water use during the year 2008, which was drier than normal years. The annual water supply demands in Table 2 vary monthly and have various return flow specifications. The hydrologic period-of-analysis in the official TCEQ WAM system Brazos data set is 1940–1997. Wurbs and Kim (2008) developed extended hydrology covering 1900–2007. This longer hydrologic period-of-analysis is particularly useful in CRM studies because frequency estimates are improved as sample size increases.

The BRA owns and operates three reservoirs and has contracted with the U.S. Army Corps of Engineers (USACE) for the water supply storage capacity of nine federal multiple-purpose reservoirs. The 14 reservoirs in the BRAC model and Table 1 account for 78% of the conservation storage capacity of the 711 reservoirs. Proctor, Waco, Squaw Creek, and Hubbard Creek are operated as individual reservoirs. The ten others are operated as a multiple-reservoir system to supply diversions at downstream sites and lakeside diversions. Multiple-reservoir release decisions in the model are on the basis of balancing storage as a percentage of capacity. Inflows are passed through reservoirs as necessary to protect downstream senior rights.

The CRM results presented in this paper focus on the five BRA reservoirs in the Little River sub-basin above the Cameron gauge shown in Fig. 2, but the entire Brazos Basin including the entire BRA system is included in the simulations. Lake Proctor is operated as an individual reservoir to supply diversions from the lake and the Leon River below the dam. Lakes Belton, Stillhouse Hollow, Georgetown, and Granger are operated as a system to supply downstream diversions from the Little River and lakeside diversions. These four reservoirs also are components of the 10-reservoir system that supplies diversions from the lower Brazos River.

Conventional Long-Term Simulation

Results from a conventional long-term simulation are presented in Figs. 3–5 and Table 3. The great temporal variability in streamflows throughout Texas is illustrated by the available flows of the Little River at the Cameron gauge, which is plotted in Fig. 3. These flows represent volumes that would have occurred during each of the 1,296 months of 1900–2007 at this site without the 14 reservoirs and associated water use included in the BRAC data set, but with the numerous smaller reservoirs and water users. Simulated storage contents of Lakes Belton, Stillhouse Hollow, Georgetown, and Granger are plotted in Fig. 4. Both inflows available to the BRA and simulated storage of Lake Proctor are plotted in Fig. 5.

End-of-June storage contents associated with specified exceedance frequencies are tabulated in Table 3 as a percentage of conservation storage capacity. The frequencies are the percentage of the 108 years of the simulation during which the end-of-June storage contents equaled or exceeded the indicated amount. The storage frequency analysis of long-term simulation results in Table 3 can be compared with CRM results that are presented later.

Lakes Belton, Stillhouse Hollow, Georgetown, and Granger are a 4-reservoir subsystem of the BRA system. Table 3 indicates that

Table 2. Annual Water Supply Diversion Targets in the BRAC Model

	Annual targets (1,000 m^3/year)			
Water supply diversion locations	Industrial	Irrigation	Municipal	Total
Lake Proctor	0	5,476	3,326	8,803
Leon River between Proctor and Belton	0	252	7,735	7,986
Lake Belton	0	0	53,324	53,324
Lake Stillhouse Hollow	0	69	33,039	33,108
Lake Georgetown	0	0	16,585	16,585
Lake Granger	0	1	3,459	3,460
Sites on Little River below lakes	3,242	263	0	3,505
Sites on Brazos above Whitney Dam	72,704	5,445	10,274	88,423
Brazos River at Bryan gauge	0	2,851	401	3,252
Brazos River at Hempstead gauge	44,347	37	0	44,385
Brazos River below Hempstead gauge	0	286	0	286
Lake Hubbard Creek	2,516	916	8,813	12,246
Lake Aquilla	0	0	7,054	7,054
Lake Somerville	0	0	4,318	4,318
Lake Limestone and Navasota River	44,493	0	223	44,723
Totals	167,309	15,597	148,550	331,457

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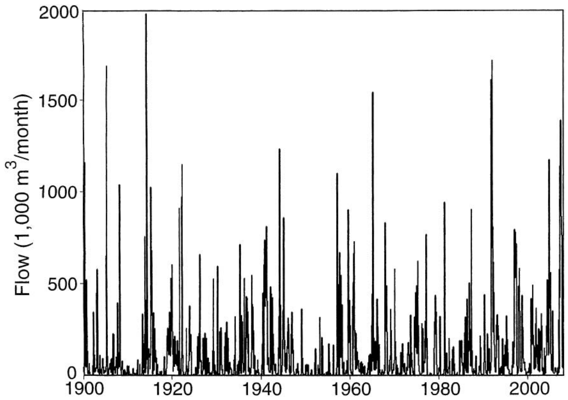

Fig. 3. Flows of the Little River at the Cameron gauge available to the Brazos River Authority

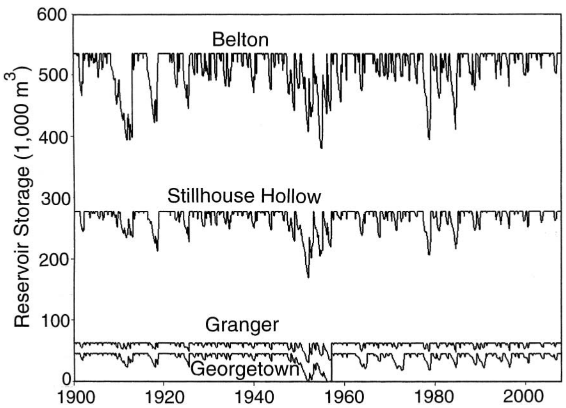

Fig. 4. Storage contents of four reservoirs

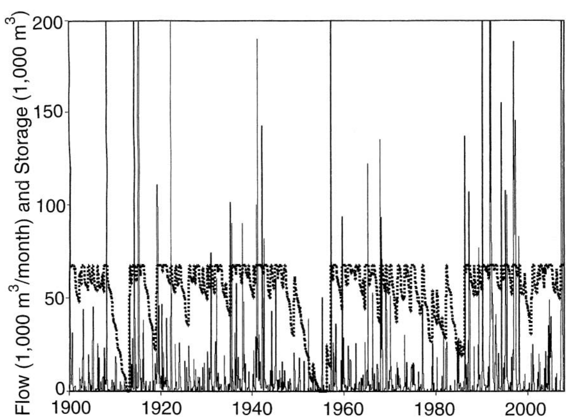

Fig. 5. Available flow (solid line) and storage (dashed line) at Proctor Reservoir

the summation of the storage contents of the four reservoirs at the end of June equaled or exceeded 83.02% of their capacity in 98% of the 108 years of the 1900–2007 hydrologic period-of-analysis. The lowest total storage level of the four-reservoir system was 82.35% of capacity. Year 2008 water demands do not result in large draw-downs in these reservoirs. Draw-downs are much more severe in Lake Proctor. The storage contents of Lake Proctor at the end of June equaled or exceeded 45.98% of capacity in 90% of the 108 years. The lake was below 45.98% of capacity at the end

Table 3. Long-Term Simulation Frequency Statistics for End-of-June Storage

Exceedance frequency (%)	End-of-June storage as percentage of capacity					
	Belton	Stillhouse	Georgetown	Granger	Total	Proctor
100	80.03	76.39	0.00	70.85	82.35	0.00
98	82.61	83.43	26.31	86.54	83.02	9.19
95	87.11	86.59	53.62	88.40	85.20	28.48
90	92.17	92.04	62.30	94.87	89.62	45.98
80	98.28	98.27	84.35	97.70	96.47	70.87
70	99.58	100.0	96.32	100.0	99.06	91.57
60	100.0	100.0	99.77	100.0	99.91	95.49
50	100.0	100.0	100.0	100.0	100.0	98.79
40	100.0	100.0	100.0	100.0	100.0	100.0

of June in 10% of the years. This difference between Proctor and the 4-reservoir system is significant in discussing CRM.

Storage Frequency Relationships on the Basis of the Equal-Weight CRM Strategy

The input data set for the CRM simulations are the same as used in the long-term simulation of the preceding section, except that the 1900–2007 hydrologic period-of-analysis is divided into 107 annual hydrologic simulation sequences. An annual cycle starting at the beginning of July is adopted for this illustrative case-study presentation. Exceedance frequencies are computed for the storage three months and 12 months later at the end of September and June. CRM results using the equal-weight method are presented in this section, and those using the probability-array method are presented in the next section. CRM results presented in this paper were derived from four executions of the simulation model with the beginning-of-July storage in each of the 14 reservoirs set at 100%, 75%, 50%, and 25% of water supply storage capacity.

A storage frequency relationship for Lake Proctor for beginning-of-July storage that was developed by using the equal-weight option is presented in Table 4. Reservoir storage contents three and 12 months later, which corresponded to the exceedance frequencies listed in the first column of Table 4, are tabulated as a percentage of water supply storage capacity. The frequencies are percentages of the 107 annual simulation sequences for which

Table 4. Storage-Frequency Relationship for Lake Proctor on the Basis of the Equal-Weight Method

Exceedance frequency (%)	Beginning-of-July storage as percentage of capacity							
	100	75	50	25	100	75	50	25
	End-of-September storage (% capacity)				End-of-June storage (% capacity)			
100	78.32	56.27	35.63	13.59	58.10	39.76	22.03	3.25
98	79.73	56.97	36.09	13.86	62.74	41.46	23.27	3.95
95	81.12	58.11	36.84	14.32	68.11	43.23	24.52	4.68
90	82.68	59.11	37.49	14.73	70.59	46.35	26.60	5.70
80	83.96	60.11	38.12	15.10	78.22	48.98	28.23	6.60
70	84.86	60.78	38.55	15.37	94.55	49.47	28.69	6.89
60	85.00	61.35	38.91	15.61	96.11	59.31	28.88	7.01
50	85.05	61.41	38.97	15.64	99.51	80.69	35.22	7.12
40	88.17	61.49	39.02	15.67	100.0	96.08	87.71	30.50
30	90.43	61.54	39.05	15.69	100.0	100.0	99.06	76.38
20	93.29	62.77	39.45	15.93	100.0	100.0	100.0	100.0
10	100.0	64.92	40.54	16.59	100.0	100.0	100.0	100.0
0	100.0	100.0	42.52	17.86	100.0	100.0	100.0	100.0

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the end-of-September or end-of-June storage volumes equaled or exceeded the indicated amount.

Table 4 indicates that with beginning-of-July storage at 50% of capacity in each of the 14 reservoirs, the end-of-September storage in Proctor equaled or exceeded 36.84% capacity in 95% of the 107 annual simulation sequences. This can be interpreted as a 95% probability that the storage level in Proctor will be equal to or greater than 36.84% of capacity by the end of September if the storage level at the beginning of July is 50% of capacity.

Likewise, a storage frequency table for the total storage in the four-reservoir system for the given beginning-of-July storage levels is presented in Table 5. Storage volume is expressed as a percentage of the total water supply storage capacity in the four reservoirs of 919 million m^3. The frequencies are the percentages of the 107 annual simulation sequences for which the summation of the end-of-September or end-of-June storage volumes of the four reservoirs equaled or exceeded the indicated percentage of capacity.

The equal-weight strategy is straightforward to apply and interpret. Storage frequency statistics developed on the basis of this method provide valid estimates of the likelihood that various storage levels will be equaled or exceeded in the future given the preceding storage levels. However, with the equal-weight option, the probabilities associated with each of the 107 hydrologic sequences in the frequency analysis are the same regardless of whether initial reservoir storage levels are specified to be 100, 75, 50, or 25% of capacity. The concept of hydrologic persistence implies that dry hydrologic conditions are more likely to follow dry conditions than wet conditions. Reservoir storage levels of 25% of capacity indicate drier previous hydrologic conditions than do storage levels at 75% capacity. Therefore, the set of probabilities associated with the 107 hydrologic sequences perhaps should be different in CRM with different initial reservoir storage conditions. This is the motivation for the alternative probability-array method.

Storage Frequency Relationships on the Basis of the Probability Array CRM Strategy

The probability-array method assigns varying probabilities to each of the hydrologic simulation sequences on the basis of relating the available flow volume (Q_S) for a specified number of months to preceding storage volume (S) by using an equation with regression coefficients a and b:

$$Q_s = ae^{(S/b)} \tag{1}$$

The exponential form of the regression equation [Eq. (1)] was adopted for the case study primarily on the basis of comparing correlation coefficients for the alternative forms of linear and nonlinear regression options selected for inclusion in WRAP. Other metrics such as percent bias and root-mean-square error also may be used for selecting between alternative forms of regression.

Correlation Analyses

The relative advantage of the probability-array versus the equal-weight strategy depends on the degree of correlation between naturalized or available flow and the preceding simulated reservoir storage volume. With negligible correlation, the equal-weight method may be more accurate. A significant degree of correlation between flow and storage implies that the more complex probability-array approach is likely worthwhile. If the probability-array option is adopted, a decision is required regarding the number of months of flow to sum in relating the available flow volume to the preceding storage volume. Correlation statistics are useful for making these modeling decisions.

Standard linear correlation coefficients (r) and Spearman rank correlation coefficients for the 1900–2007 monthly available flow and storage volumes that result from the long-term simulation are tabulated in Table 6 along with the correlation coefficients associated with Eq. (1). The Spearman rank correlation coefficient is the linear correlation coefficient computed for the relative ranks of the storage and flow volumes rather than the actual volumes. Flow volumes for 1, 2, 3, 6, 9, and 12 months are correlated with preceding storage volumes. Flows at Proctor dam are correlated with storage in Lake Proctor. Flows at the Cameron gauge on the Little River are correlated with the total storage in Lakes Belton, Stillhouse Hollow, Granger, and Georgetown. The correlation coefficients in Table 6 are relatively small positive numbers, which indicate a noticeable though small correlation between flow during a time period and the preceding reservoir storage. The correlation coefficients generally decrease with increases in the number of months over which flow volumes are summed.

Probability Array Computational Methodology

The CRM methodology is outlined in Fig. 1 from the perspective of options selected by the WRAP user in applying the generalized modeling system. The probability-array-based methodology is comprised of three sequential sets of computations that are described in this section. First, a storage-flow-frequency (SFF) array is developed from the results of a long-term simulation. Next, the SFF is used to develop an incremental probability array. Finally, the incremental probability array is applied in frequency and

Table 5. Storage-Frequency Relationship for the Four-Reservoir System on the Basis of the Equal-Weight Method

Exceedance frequency (%)	Beginning-of-July storage as percentage of capacity							
	100	75	50	25	100	75	50	25
	End-of-September storage (% capacity)				End-of-June storage (% capacity)			
100	89.74	65.83	47.93	18.70	82.35	59.10	36.14	13.88
98	90.32	67.29	48.30	18.88	86.83	63.72	39.95	16.57
95	91.07	67.94	48.89	19.23	89.60	65.85	42.83	20.26
90	92.03	68.75	49.53	19.54	94.87	75.03	52.08	28.77
80	92.42	69.05	49.98	19.94	97.77	87.29	64.99	41.95
70	93.93	70.57	51.39	21.14	99.05	93.60	72.54	51.34
60	94.68	71.22	52.34	22.11	99.89	97.99	83.19	60.71
50	95.94	74.27	55.64	24.87	100.0	99.51	93.69	80.56
40	97.32	76.63	58.22	27.07	100.0	100.0	99.81	94.24
30	98.07	80.36	62.61	31.11	100.0	100.0	100.0	99.89
20	99.35	88.31	71.37	38.38	100.0	100.0	100.0	100.0
10	99.83	95.77	82.81	48.53	100.0	100.0	100.0	100.0
0	100.0	98.60	88.03	63.98	100.0	100.0	100.0	100.0

Table 6. Correlation Coefficients for Beginning-of-July Storage Versus Available Flow

Reservoir or system	Form of relationship	Number of months of flow volume					
		1	2	3	6	9	12
Lake Proctor	Linear	0.113	0.121	0.155	0.169	0.145	0.112
Lake Proctor	Spearman	0.440	0.426	0.415	0.289	0.251	0.106
Lake Proctor	Exponential	0.400	0.384	0.368	0.139	0.229	0.105
4-Lake System	Linear	0.182	0.168	0.181	0.211	0.167	0.112
4-Lake System	Spearman	0.612	0.585	0.568	0.336	0.225	0.120
4-Lake System	Exponential	0.540	0.574	0.569	0.312	0.349	0.118

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reliability analyses to assign probabilities to each of the simulation sequences.

In general, natural streamflow volume may be the summation of flows at any number of locations over any number of months. Simulated storage may include any number of reservoirs. In the simplest case, flow volume at one location is related to storage volume in one reservoir.

An SFF array developed from the results of a long-term simulation relates the flow ratio $Q_{\%}$ [Eq. (2)] to exceedance probability on the basis of either the log-normal probability distribution or the Weibull relative frequency formula:

$$Q_{\%} = \frac{Q}{Q_S} = 100\% \tag{2}$$

where Q = naturalized or available flow volume over one or more months; Q_S = corresponding volume from a regression equation [Eq. (1)] that reflects the preceding storage volume; and $Q_{\%}$ = flow as a percentage of the expected value of flow conditioned on preceding storage as modeled by a regression equation developed from the results of a long-term simulation.

An SFF array is developed from the results of a long-term simulation as follows:

1. Naturalized or available flow and preceding reservoir storage volumes are read from the results of the long-term simulation and summed to develop the data set for the regression,
2. Regression analyses are performed to relate available flow volume during one or more months at one or more control points to the preceding storage volume in one or more reservoirs by using one of four alternative regression equations,
3. The expected value of flow conditioned on storage is computed for each simulation sequence by using the regression equation derived in step 2. The corresponding values of $Q_{\%}$ in percent are determined with Eq. (2), and
4. The SFF array is developed by assigning exceedance probabilities to $Q_{\%}$ by using the log-normal probability distribution, or alternatively by using the Weibull formula.

After creating the SFF array on the basis of results from a long-term simulation, an incremental probability array is developed from the SFF array. The incremental probability array is comprised of the probabilities P_S assigned to each of the hydrologic sequences and corresponding CRM simulations. The incremental probabilities P_S for all simulation sequences sum to 1.0. Probabilities are assigned to each of the CRM simulations as follows:

1. A WRAP post-simulation program organizes the user-specified initial reservoir storage contents and naturalized or available flow volumes obtained from the results of executing the simulation model in CRM mode. The initial storage volumes of specified reservoirs are summed to obtain the total storage amounts used for developing the incremental probability array. Naturalized or available flows during specified months at specified control points are summed to obtain the total flow amounts used in the computations,
2. The expected value of flow Q_S conditioned on the preceding storage is computed for each simulation sequence by using the regression equation from step 2 for developing the SFF array. The corresponding values of the flow ratio $Q_{\%}$ are determined with Eq. (2),
3. $Q_{\%}$ for each sequence is combined with the SFF array by using linear interpolation to obtain an exceedance probability for each CRM simulation sequence, and
4. The exceedance probabilities are ranked in order and converted to incremental probabilities P_S as the difference between the two exceedance probabilities.

This process results in a probability array that assigns a P_S to each of the hydrologic sequences and associated short-term CRM simulations. The incremental probability array with P_S values summing to 1.0 is used in the frequency computations, which are on the basis of the relative frequency. The P_S values are incorporated in the frequency computations by counting each simulation sequence multiple times with the count totaling to 1,000,000, which is an arbitrarily selected large integer number. The number of times (N_S) that each short-term simulation is repeated in the frequency count is proportional to P_S as follows:

$$N_S = 1{,}000{,}000\ P_S \tag{3}$$

The N_S of Eq. (3) are applied in basically the same manner in either water-supply diversion reliability, regulated-flow frequency, or reservoir-storage frequency analyses. The Brazos case study in this paper has 107 hydrologic sequences applied in 107 simulations, each of which is assigned an N_S. Each end-of-September or end-of-June storage volume of each reservoir computed in the 107 simulations is assigned an incremental probability of $N_S/1{,}000{,}000$. Incremental probabilities are accumulated to obtain the exceedance probabilities assigned to each of the 107 storage volumes.

Case Study Probability-Array-Based Analysis and Results

Storage frequency tables developed with the probability-array strategy are presented in Tables 7 and 8, which are identical in format and interpretation as Tables 4 and 5. The simulations are the same for the equal-weight and probability-array methods. Frequency analyses provide the same information with either computation strategy although the computational methods differ.

The SFF array is developed from the results of the long-term 1900–2007 simulation for which storage volumes are plotted in Figs. 4 and 5. Flows at the Cameron gauge and Proctor Dam are plotted in Figs. 3 and 5. Correlation coefficients for the preceding storage versus available flow volume are presented in Table 6. The SFF array for the four-reservoir system relates the total beginning-of-July storage in the four reservoirs in each of 107 years to the July-September flow volume of the Little River at the Cameron gauge. Likewise, the SFF array for Lake Proctor relates beginning-of-July storage to the following three-month available

Table 7. Storage-Frequency Relationship for Lake Proctor on the Basis of the Probability-Array Method

Exceedance Frequency (%)	Beginning-of-July storage as percentage of capacity							
	100	75	50	25	100	75	50	25
	End-of-September storage (% capacity)				End-of-June storage (% capacity)			
100	78.32	56.27	35.63	13.59	58.10	39.76	22.03	3.25
98	80.59	56.34	36.09	13.60	62.28	41.76	24.21	4.38
95	81.17	58.00	36.77	13.92	67.40	43.12	24.21	4.38
90	82.69	59.14	37.08	14.29	70.88	46.17	26.51	4.71
80	83.97	59.64	37.83	14.93	82.45	48.97	28.33	6.77
70	84.97	60.69	37.83	14.93	94.70	49.47	28.82	6.96
60	85.05	61.35	38.39	14.93	96.19	53.47	29.17	6.99
50	87.62	61.43	38.93	15.13	98.79	71.70	31.62	7.01
40	90.13	61.50	39.00	15.61	100.0	94.53	42.47	8.61
30	93.43	61.53	39.03	15.68	100.0	98.75	63.86	8.61
20	100.0	61.96	39.05	15.69	100.0	100.0	98.79	8.61
10	100.0	63.63	40.10	15.69	100.0	100.0	100.0	72.57
0	100.0	100.0	42.52	17.86	100.0	100.0	100.0	100.0

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Table 8. Storage-Frequency for Four-Reservoir System on the Basis of the Probability-Array Method

Exceedance Frequency (%)	Beginning-of-July storage as percentage of capacity							
	100	75	50	25	100	75	50	25
	End-of-September storage (% capacity)				End-of-June storage (% capacity)			
100	89.74	65.83	47.93	18.70	82.35	59.10	36.14	13.88
98	90.69	65.89	47.93	18.70	86.79	63.51	39.68	16.34
95	91.25	66.08	48.09	18.78	90.41	65.37	42.22	19.02
90	92.37	66.65	48.57	19.02	95.05	73.03	50.36	27.36
80	93.60	67.16	48.79	19.43	98.34	83.79	60.53	37.63
70	93.98	67.80	49.57	19.56	99.44	89.65	68.04	45.27
60	95.69	68.02	49.70	19.61	100.0	96.43	81.26	60.32
50	97.17	68.04	49.72	19.62	100.0	98.65	84.68	62.30
40	97.95	69.19	50.86	20.64	100.0	99.97	94.03	81.22
30	98.92	71.17	53.39	22.66	100.0	100.0	100.0	98.57
20	99.90	76.00	58.79	27.67	100.0	100.0	100.0	100.0
10	100.0	87.68	63.63	39.40	100.0	100.0	100.0	100.0
0	100.0	98.60	88.03	63.98	100.0	100.0	100.0	100.0

flow volume at the Leon River at Proctor Dam. The choice of number of months for which flow volumes are summed is on the basis of correlation analyses and judgment. An exponential form of regression equation [Eq. (1)] was selected to relate three-month flow volume Q_S to the preceding beginning-of-July storage volume S.

WRAP includes options of using either the log-normal probability distribution or relative frequency as reflected in the Weibull formula for assigning exceedance probabilities to the flow ratio $Q_{\%}$ defined by Eq. (2). The Weibull formula [Eq. (4)] was adopted for the results presented in Tables 7 and 8:

$$P = \frac{m}{N + 1} \tag{4}$$

where P = exceedance probability; and m = rank $(1, 2, \ldots, N = 107)$. The sample size of 107 is judged to be adequate to define a frequency distribution with Eq. (4). The log-normal option may be advantageous for a shorter hydrologic period of analysis.

Comparative Discussion of the Equal-Weight versus Probability-Array Methods

Future reservoir storage consists of water from two sources: (1) the volume currently in storage, and (2) hydrology represented by stream inflows and net reservoir evaporation rates during the future period of interest. The relative importance of these two sources depends on their relative magnitude. Initial reservoir storage is specified the same with either the equal-weight or probability-array options. The goal of the probability-array option is to improve probability estimates assigned to the hydrologic sequences. With longer simulation periods, such as 12 versus 3 months, hydrology plays a greater role relative to initial storage in determining future storage, but the correlation between available flow and preceding storage decreases.

The degree of correlation between available streamflow volume and preceding simulated storage volume is fundamental for considering improvements in accuracy to be achieved by adopting the probability-array rather than the equal-weight method. Analyses performed for sites throughout the Brazos River Basin indicate that correlations are not high at any of the reservoirs, although the degree of correlation varies between reservoirs. The correlation is small but non-negligible in most cases, including the reservoirs included in the illustrative case study presented in this paper.

Actual current storage contents are known in real-time water management applications. In planning studies, ranges of preceding storage levels are considered in the CRM analyses. The specified initial storage may not always be related to hydrology. For example, a reservoir might be drained for purposes of performing maintenance or rehabilitation construction on the dam.

The alternative initial storage contents of 100, 75, 50, and 25% of capacity adopted for the CRM analyses were selected arbitrarily to cover the range from full to severely drawn-down. Four of the reservoirs are completely emptied, but the ten others do not drop below 25% of capacity during the 1900–2007 long-term simulation. From a basin-wide perspective, initial CRM storage of 25% capacity represents a drought more severe than any drought occurring during 1900–2007.

Consistency between the range of storage volumes reflected in the CRM as compared with the long-term simulation is an issue in applying either method, but it is particularly significant with the probability array method. The SFF array should be developed from the results of a long-term simulation with storage draw-downs that are reasonably representative of the draw-downs that occur in the CRM simulations. The plot of Fig. 5 and storage frequency statistics in Table 3 indicate that Lake Proctor experiences the full range of storage levels from empty to full during the 1900–2007 long-term simulation. However, Lakes Belton, Stillhouse, and Granger do not experience severe draw-downs in the long-term model. Thus, the SFF array-based CRM analyses for the four-reservoir system with initial storage at 25% and 50% of capacity are highly questionable. With either Proctor or the four-reservoir system, the probability-array analyses are most valid for initial storage conditions of 100% or 75% capacity, and least valid for initial storage conditions of 25% capacity.

With the preceding storage contents set at 100% or 75% of capacity, the results in Tables 4 and 7 and Tables 5 and 8 are approximately the same with the equal-weight and probability-array methods. With initial storage set at 50% or 25% of capacity, the differences are significant. With the initial storage set at 100% of capacity, the future storage volumes associated with a specified exceedance frequency computed on the basis of the equal-weight option are generally slightly smaller than the storage computed with the probability-array option. As the initial storage is dcreased, the volumes computed on the basis of the equal-weight option become larger than those with the probability array option. The differences between results with the two alternative methods generally tend to be greater for the 12-month than the 3-month simulation, which reflects the greater role of stream inflow versus initial storage.

With the equal-weight method, each simulation is weighted the same, which is equivalent to assigning a probability of $1/107$ to each of the 107 simulations. Equal-weight option predictions of future storage levels for given exceedance frequencies logically should be conservatively low for high initial storage such as 100% of capacity. With low initial storage such as 25% of capacity, future storage volumes on the basis of the equal-weight option would be expected to be generally high.

The probability-array option is designed to model hydrologic persistence as reflected in the initial storage contents. With the initial storage contents set near capacity, the probability array option assigns higher probabilities to high flows and lower probabilities to low flows than does the equal-weight option, which should improve accuracy. Likewise, with a low initial storage, higher probabilities are assigned to low flows and lower probabilities to high flows.

Conclusions

The WRAP/WAM system is routinely applied in planning studies and preparation and evaluation of water-right permit applications in Texas. The range of modeling applications is growing. WRAP has been expanded to incorporate CRM capabilities designed for short-term analyses to support operational planning and drought management. CRM is an extension of WRAP modeling capabilities that requires only minimal modifications to existing input data sets and preserves all existing capabilities for simulating complex water management practices.

The equal-weight option for CRM is simple to understand and apply. Unlike the probability-array-based alternative, the equal-weight method does not require the model user to choose between an assortment of modeling options, all of which affect the CRM results. Storage frequency statistics developed on the basis of the equal-weight method provide valid estimates of probabilities of various storage volumes that are equaled or exceeded in the future, given the preceding storage levels.

However, hydrologic persistence is a significant issue. Improved accuracy of likelihood estimates can be achieved by using the probability-array method in appropriate situations. The probability-array strategy improves the accuracy of frequency and reliability metrics by using reservoir storage as an index of past hydrologic conditions. The probability-array option is generally advantageous over the equal-weight method if the naturalized flow is significantly correlated with preceding storage and the long-term simulation produces a range of storage levels comparable to the initial levels specified in the CRM analysis.

Capabilities for supplying water needs over the next several months depend on water presently available in storage, future stream inflows, and other factors. Future inflows play a greater role in determining storage 12 months into the future rather than three months, but the storage-flow correlation decreases. A greater reliance on future inflows increases the importance of the improvements in accuracy achieved by the probability-array method. However, storage-flow correlation and associated utility of the probability-array option relative to the equal-weight option decrease as the analysis looks further into the future. Because SFF arrays are site specific, the equal-weight option may be advantageous for applications involving numerous reservoirs and water users scattered over a large river basin.

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